Regulation, Supervision, and Bank Risk-Taking

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Abstract

This paper presents a model of a bank that chooses the risk of its investment portfolio and a supervisor that collects noisy information on the banks' solvency and, based on this information, may decide on its early liquidation. The paper characterizes the liquidation decision of the supervisor and the risk-taking decision of the bank. In line with recent empirical literature, the paper shows that supervision is effective in ameliorating the bank's risk-taking incentives, and that higher supervisory noise may lead to lower risk-taking. The paper also analyzes the joint effect of regulation and supervision on risk-taking and social welfare.

JEL Classification: G21, G28, D02.

Keywords: Bank risk-taking, bank supervision, bank regulation, capital requirements, deposit insurance, bank resolution, bank liquidation.

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1 Introduction

Bank supervision, unlike bank regulation, has not been until recently the subject of much academic interest. As stated by Eisenbach et al. (2016), regulation involves the establishment of rules under which banks operate, while supervision involves the assessment of safety and soundness of banks through monitoring, and the use of this information to request corrective actions. In contrast to regulation, that is based on verifiable information, supervisory actions are (partly) based on nonverifiable information.

In recent years a number of empirical papers on bank supervision (summarized in the literature review below) have been published, showing that supervision has a disciplining effect on bank risk-taking. The purpose of this paper is to construct a theoretical model of the interaction between a bank and a supervisor that can account for these empirical results. The model features a bank that privately chooses the risk of its investment portfolio and a supervisor that collects nonverifiable information on the solvency of the bank and, based on this information, may decide on its early liquidation. The paper characterizes the liquidation decision of the supervisor and the risk-taking decision of the bank. The main result is that, compared to a laissez-faire situation, and for any quality of the supervisory information, supervision reduces the bank's risk-taking.

The model has three dates. At $t = 0$ the bank raises a unit amount of insured deposits and invests them in a risky asset that yields a *liquidation return* L at $t = 1$, and a *final return* R at $t = 2$, if not liquidated by the supervisor at $t = 1$. Both L and R are random variables with a variance that depends on the bank's choice of risk σ at $t = 0$. The supervisor observes at $t = 1$ a *noisy signal* $s = R + \varepsilon$ of the final return R of the bank's investment, which is interpreted as the outcome of bank supervision. Given the key role of judgement in bank supervision, the signal s is assumed to be nonverifiable. The variance of the noise term ε is proportional to a parameter θ, whose inverse measures the quality (technically the precision) of the supervisory information.

To make the model tractable, I assume that the probability distribution of asset returns and supervisory signals is normal. And to guarantee an interior choice of risk, I assume that deviating from a reference value of risk $\overline{\sigma} > 0$ entails an increasing and convex cost for the bank. The interpretation is that $\overline{\sigma}$ characterizes the business model of the bank, which is risky, and that deviating from it is costly. The assumptions of limited liability and deposit insurance imply that in a laissez-faire situation the bank chooses a level of risk above the reference value $\overline{\sigma}$.

Following standard supervisory practice, I next assume that the supervisor uses a *failing or likely to fail rule* whereby the bank is liquidated at $t = 1$ when the conditional expected final return $E(R \mid s)$ is smaller than the unit face value of the deposits. The condition $E(R \mid s) < 1$ yields a threshold $\widehat{s}$ such that the bank is liquidated when $s < \widehat{s}$.

To analyze the bank's choice of risk under supervision, note that the bank will get a positive payoff when (i) the final return R is greater than the unit face value of deposits, and (ii) the signal s observed by the supervisor is greater than or equal to the threshold $\widehat{s}$. Using the properties of truncated bivariate normal distributions (summarized in Appendix A), one can show that there is an analytical expression for the bank's expected payoff function, which can be used to derive the bank's choice of risk. I show that, compared to the laissez-faire situation and for any quality of the supervisory information, *supervision is effective in ameliorating the bank's risk-shifting incentives.*

A result that follows from this analysis is that the relationship between the variance of the noise parameter θ in the supervisory signal and the bank's choice of risk is U-shaped, first decreasing and then increasing. In particular, for a sufficiently high quality of the supervisory information (low θ), increases in θ lead to lower risk-taking. This appears to be puzzling: why higher noise would improve incentives? To explain the result note that an increase in the variance of the supervisory noise has two effects. First, the supervisor reacts to the lower quality of its information by reducing the liquidation threshold $\widehat{s}$. Second, a higher variance of the supervisory noise makes it more likely that large negative values of the noise ε in the supervisory signal s are realized. The first effect tends to increase the risk σ chosen by the bank, while the second effect goes in the opposite direction. One can show that the second effect dominates when θ is small. Arguably, when the supervisor has access to confidential bank information, it is reasonable to assume that the value of θ should not be too large.

Hence, the conclusion is that, in the relevant range, *a larger variance of the noise in the supervisory signal is conducive to lower risk-taking.*

Interestingly, Agarwal et al. (2024) find evidence in line with this prediction of the model. Using data on the supervisory ratings of US banks, they show that bank supervisors exercise significant personal discretion, which introduces noise into their ratings. This leads to "a large and persistent causal impact on future bank capitalization and supply of credit, leading to volatility and uncertainty in bank outcomes, and a *conservative anticipatory response by banks*" (my italics).

The paper also analyzes the effect of bank capital regulation, in the form of a requirement to fund at least a fraction of its investment with costly equity capital, and its interaction with bank supervision. The results show that both regulation and supervision are effective in reducing the bank's risk-taking incentives, but that capital requirements are more powerful tools.

The paper concludes with a welfare analysis of bank regulation and supervision under the assumptions that bank supervision is costless and that deposit insurance is funded with lump sum taxation. The results show that supervision can has a positive effect on social welfare, especially when capital requirements are low, but that bank capital regulation is a better tool for addressing the bank's risk-taking incentives.[1]

Literature review A growing empirical literature studies the effects of bank supervision on banks' lending and risk-taking decisions. Research with US data starts with Agarwal et al. (2014). They find that federal supervisors are systematically tougher than state supervisors, which translates into lower failure rates. Hirtle et al. (2020) show that banks receiving greater supervisory attention hold less risky loan portfolios and adopt more conservative reserving practices. Kandrac and Schlusche (2021) find that reductions in supervisory capacity leads to costlier failures, while Costello et al. (2019) show that stricter supervisors are more likely to enforce restatements of banks' call reports. Eisenbach et al. (2022) develop and

[1] It should be noted that the results on the effect of bank supervision (but not those on the effect of bank regulation) rely on a numerical solution of the model for a particular set of parameter values, but are expected to be true in general.

estimate a structural model in which supervisory attention in creases with bank size and riskiness. Beyhaghi et al. (2024) show that tougher-than-expected bank ratings lead to a decrease in risk-taking, profitability, and loan growth, and Correia et al. (2025) show that exogenous variation in supervisory strictness during the Global Financial Crisis increased loss recognition, reduced dividends, and an increased the likelihood of bank closure.

Complementary evidence is provided with non-US data. Passalacqua et al. (2019) find that inspected banks in Italy are more likely to reclassify loans as non-performing, and that this reclassification leads to a contraction in lending. Haselmann et al. (2022) and Altavilla et al. (2024) analyze the effects of supranational versus national supervision, showing that the tougher supranational supervision leads to higher capital and reduced lending to riskier firms. Kok et al. (2023) show that supervisory scrutiny associated to stress testing has a disciplining effect on bank risk-taking. Bonfim et al. (2023) find that on-site inspections of Portuguese banks reduced refinancing to zombie firms. Abbassi et al. (2024) show that German banks subject to the 2013 asset quality review reduced lending to riskier firms. Finally, Degryse et al. (2025) show that automatic supervisory alerts used by the Central Bank of Brazil increased provisions for risky loans and reduced lending to riskier borrowers.

Taking together, this empirical literature provides *strong and consistent evidence that increased supervisory intensity leads to a significant reduction in bank risk-taking.* The question that they beg is what is the mechanism whereby supervision has this effect on risk-taking. This is where the main contribution of this paper lies.

In contrast with the huge theoretical literature on bank regulation,[2] the literature on bank supervision is pretty thin. An early contribution is Mailath and Mester (1994). They consider a supervisor's incentives to close a bank, recognizing the opportunity cost of forgone intermediation if the bank is closed as well as the effect the supervisor's closure policy on the bank's risk-taking. They consider two objective functions for the supervisor, maximizing social welfare or minimizing closure costs, showing that even in the first case the lack of commitment power on the part of the supervisor leads to second-best outcomes.

[2]See, for example, Dewatripont and Tirole (1994), Bhattacharya (1998), Hellmann et al. (2000), and Repullo (2004), and Vives (2016).

Eisenbach et al. (2016) present a rich framework for the analysis of the interaction between a bank and a supervisor. In this framework, the bank first chooses a risk-taking action, the supervisor observes a signal about the bank's action and then chooses a corrective action. The final asset return depends on both the bank's and the supervisor's actions. It is assumed that the bank's action is costly, that the supervisor's monitoring and intervention are also costly, and that it cares about possible negative spillovers from a bank failure. Among other results, they show that riskier banks receive more supervisory attention and feature a higher level of intervention than safer banks.[3]

Repullo (2018) presents a model in which a central and a local supervisor contribute their efforts to obtain information on the solvency of a local bank, which is then used by the central supervisor to decide on its early liquidation. This hierarchical model is contrasted with the alternatives of decentralized and centralized supervision. The local supervisor has a higher bias against liquidation and a lower cost of getting local information. The paper characterizes the conditions under which hierarchical supervision is the optimal institutional design. In contrast with this current paper, it is assumed that the structure of supervision does not affect bank risk-taking, which is taken to be exogenous. But both papers assume that the probability distribution of the liquidation return, the final return, and the supervisory signal is normal, taking advantage of the nice properties of normally distributed random variables.[4]

Structure of the paper Section 2 presents the model setup. Section 3 characterizes the bank's choice of risk in the absence of regulation or supervision. Sections 4 and 5 analyze, respectively, the effect on risk-taking of bank capital regulation and supervision. Section 6 considers the interaction between regulation and supervision. Section 7 presents the welfare analysis, and Section 8 concludes. Appendix A summarizes some useful properties of the bivariate normal distribution and Appendix B contains the proofs of the propositions.

[3] Eisenbach et al. (2016) is the working paper version of Eisenbach et al. (2022). One interesting difference between the two papers is that in the working paper the signal of the supervisor is about the bank's initial action, while in the published paper the signal is about future distress.

[4] Other theoretical papers that have analyzed the institutional structure of bank supervision are Colliard (2016), Calzolari et al. (2019), and Carletti et al. (2020). This last paper is especially interesting because it considers effect of supervisory architecture on bank risk-taking.

2 Model setup

Consider an economy with three dates ($t = 0, 1, 2$) and two risk-neutral agents: a bank and a bank supervisor. The bank raises a unit amount of deposits at $t = 0$, and invests them in an asset that has a random *final return* R at $t = 2$. The asset can be liquidated at $t = 1$, in which case it yields a random *liquidation return* L. Deposits are insured by a deposit insurer that charges a flat premium normalized to zero. The deposit rate is also normalized to zero.

Asset returns are normally distributed with

$$\begin{bmatrix} L \\ R \end{bmatrix} \sim N\left(\overline{R}\begin{bmatrix} a \\ 1 \end{bmatrix}, \sigma^2 \begin{bmatrix} b & c \\ c & 1 \end{bmatrix}\right), \tag{1}$$

where $\overline{R} > 1$, $0 < a < 1$, $0 < b < 1$, and $c > 0$.

Thus, the expected final return $E(R) = \overline{R}$ is greater than the unit face value of the deposits, and it is also greater than the expected liquidation return $E(L) = a\overline{R}$. This means that, in the absence of information at $t = 1$, liquidation would be inefficient. Moreover, the final return R has a higher variance than the liquidation return L, and both returns are positively correlated. These assumptions are very reasonable: uncertainty tends to increase with the passage of time, and the liquidation value of a financial asset (say a loan portfolio) tends to move in line with the final value of the asset. Note that to ensure that the covariance matrix is positive-definite b must be greater than c^2, so $0 < b < 1$ implies $c < 1$.

I assume that pararameter values are such that when the final return equals the face value of the deposits, that is when $R = 1$, the conditional liquidation value satisfies

$$E(L \mid R = 1) < 1,$$

By the properties of normal distributions (see (12) in Appendix A) this condition simplifies to

$$R^* = \frac{(a - c)\overline{R}}{1 - c} < 1. \tag{2}$$

The bank chooses at $t = 0$ the risk σ its portfolio. It is assumed that deviating from a reference value $\overline{\sigma} > 0$ entails a nonpecuniary cost for the bank given by

$$c(\sigma) = \frac{\gamma}{2}(\sigma - \overline{\sigma})^2, \tag{3}$$

where $\gamma > 0$.[5] The interpretation is that $\overline{\sigma}$ characterizes the business model of the bank, and that deviating from it (in either direction) is costly.

3 Laissez-faire

This section characterizes the behavior of the bank in the absence of regulation or supervision. In this case, the objective function of the bank, denoted $v(\sigma)$, is to maximize its expected payoff at $t = 2$ net of the cost of risk-taking, which gives

$$v(\sigma) = \pi(\sigma) - c(\sigma),$$

where

$$\pi(\sigma) = E[\max\{R - 1, 0\}]$$

is the bank's expected payoff.

By assumption (1) we have $R \sim N(\overline{R}, \sigma^2)$, so by the properties of truncated normal distributions (see (13) in Appendix A) we get

$$\pi(\sigma) = E[R - 1 \mid R \geq 1] \Pr(R \geq 1) = (\overline{R} - 1)\Phi\left(\frac{\overline{R} - 1}{\sigma}\right) + \sigma\phi\left(\frac{\overline{R} - 1}{\sigma}\right),$$

where $\phi(x)$ and $\Phi(x)$ are, respectively, the density function and the cumulative distribution function of a standard normal random variable x.

The function $\max\{R - 1, 0\}$ is convex, so by second-order stochastic dominance it follows that $\pi(\sigma)$ is increasing in σ. In particular, using the fact that $\phi'(x) = -x\phi(x)$ one can show that

$$\pi'(\sigma) = \phi\left(\frac{\overline{R} - 1}{\sigma}\right) > 0.$$

Subtracting from the expected payoff $\pi(\sigma)$ the convex cost of risk-taking $c(\sigma)$ allows us to find an interior solution to the maximization of the bank's objective function $v(\sigma)$.

[5] The cost $c(\sigma)$ is assumed to be nonpecuniary to ensure tractability. Note that increases in σ below $\overline{\sigma}$ increase the expected final return of the investment net of the cost of risk-taking, $\overline{R} - c(\sigma)$, so over this range there is a risk-return trade-off.

Proposition 1 *The bank's maximization problem has a solution σ^* characterized by the first-order condition*

$$\phi\left(\frac{\overline{R}-1}{\sigma^*}\right) = \gamma(\sigma^* - \overline{\sigma}). \tag{4}$$

Since the left-hand side of (4) is positive, it must be the case that $\sigma^* > \overline{\sigma}$. Hence, under laissez-faire the bank will have an incentive to increase the risk of its investment above the reference value $\overline{\sigma}$.

Figure 1 plots the bank's expected payoff $\pi(\sigma)$ and objective function $v(\sigma)$, showing the bank's choice of risk σ^* in the absence of regulation or supervision.[6]

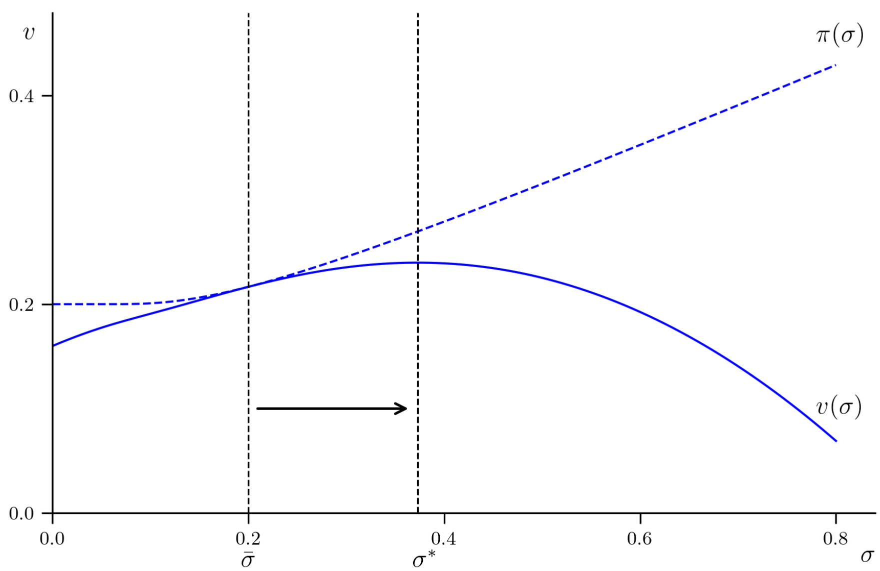

Figure 1. Risk-taking in laissez-faire

This figure plots the bank's expected payoff and objective function, showing the bank's excess risk-taking in the absence of regulation or supervision.

[6] The following parameter values are used in all the figures: $\overline{R} = 1.2$, $a = 0.75$, $b = c = 0.5$, $\overline{\sigma} = 0.2$, and $\gamma = 2$. These values are not intended to provide a calibration of the model, since they are chosen to facilitate the graphical representation of the qualitative results of the paper.

Differentiating the first-order condition (4) yields the following comparative statics results

$$\frac{d\sigma^*}{d\gamma} = \frac{\sigma^* - \overline{\sigma}}{\frac{\partial}{\partial\sigma}\left[\phi\left(\frac{\overline{R}-1}{\sigma}\right) - \gamma(\sigma - \overline{\sigma})\right]\Big|_{\sigma=\sigma^*}} < 0,$$

$$\frac{d\sigma^*}{d\overline{R}} = -\frac{\frac{1}{\sigma}\phi'\left(\frac{\overline{R}-1}{\sigma}\right)}{\frac{\partial}{\partial\sigma}\left[\phi\left(\frac{\overline{R}-1}{\sigma}\right) - \gamma(\sigma - \overline{\sigma})\right]\Big|_{\sigma=\sigma^*}} < 0,$$

where we have used that $\sigma^* - \overline{\sigma} > 0$, $\overline{R} - 1 > 0$, and the fact that the denominator of these expressions is negative by the second-order condition that characterizes the solution σ^*.

Thus, increases in the cost of deviating from the reference value $\overline{\sigma}$ and increases in the expected return of the asset reduce the bank's choice of risk. To the extent that the expected return $\overline{R}$ can be taken as a proxy for the bank's market power, this second result is consistent with the classical "charter value" view on the determinants of banks' risk-taking.

4 Bank capital regulation

This section analyzes the bank's choice of risk when it is subject to a regulation that requires to fund at least a fraction $\overline{k} \in (0, 1)$ of its investment with equity capital. As it is standard in the literature, I assume that capital is more costly than deposits, and denote by $\delta \geq 0$ the excess cost of capital.[7]

The bank's objective function now becomes

$$v(\sigma; k) = \pi(\sigma; k) - c(\sigma),$$

where

$$\pi(\sigma; k) = E[\max\{R - (1 - k), 0\}] - (1 + \delta)k$$

is the bank's expected payoff, $k \geq \overline{k}$ is the bank's capital, and $1 - k$ its deposit liabilities. Note that the bank shareholders incur the cost $(1+\delta)k$ regardless of whether the final return R is greater or smaller than $1 - k$.

[7] Note that with insured deposits $\delta > 0$ would not be needed.

Differentiating $\pi(\sigma;k)$ with respect to k gives

$$\frac{\partial\pi(\sigma;k)}{\partial k} = \frac{\partial}{\partial k}\int_{1-k}^{\infty}[R-(1-k)]\phi\left(\frac{R-\overline{R}}{\sigma}\right)dR-(1+\delta) = \Phi\left(\frac{\overline{R}-(1-k)}{\sigma}\right)-(1+\delta)<0.$$

Thus, the capital requirement will always be binding. Given this result, I will henceforth simply let k denote the capital requirement (instead of $\overline{k}$).

As in the laissez-faire case, by the properties of truncated normal distributions we get

$$\pi(\sigma;k) = [\overline{R}-(1-k)])\Phi\left(\frac{\overline{R}-(1-k)}{\sigma}\right)+\sigma\phi\left(\frac{\overline{R}-(1-k)}{\sigma}\right)-(1+\delta)k. \tag{5}$$

Proposition 2 *For any capital requirement k, the bank's maximization problem has a solution $\sigma^*(k)$ characterized by the first-order condition*

$$\phi\left(\frac{\overline{R}-(1-k)}{\sigma^*}\right)=\gamma(\sigma^*-\overline{\sigma}). \tag{6}$$

Note that, as in the laissez-faire case, we have $\sigma^*(k)>\overline{\sigma}$. Thus, regardless of the capital requirement k the bank will have an incentive to increase the risk of its investment above the reference value $\overline{\sigma}$.

Differentiating the first-order condition (6) gives

$$\frac{d\sigma^*(k)}{dk} = -\frac{\frac{1}{\sigma}\phi'\left(\frac{\overline{R}-(1-k)}{\sigma^*(k)}\right)}{\frac{\partial}{\partial\sigma}\left[\phi\left(\frac{\overline{R}-(1-k)}{\sigma}\right)-\gamma(\sigma-\overline{\sigma})\right]\Big|_{\sigma=\sigma^*(k)}}<0,$$

where we have used that $\overline{R}-(1-k)>\overline{R}-1>0$, and the fact that the denominator of this expression is negative by the second-order condition that characterizes the solution $\sigma^*(k)$. Thus, an increase in the capital requirement k reduces the bank's risk-taking.

Figure 2 shows the bank's expected payoff and objective function without (in blue) and with (in green) a capital requirement k.[8] The introduction of a capital requirement shifts down both functions, reflecting the increase in the cost of funding and the reduction in the deposit insurance subsidy. Moreover, due to a "skin in the game" effect, the bank chooses a lower level of risk compared to the laissez-faire case.

[8] In this figure I set the capital requirement $k=0.1$ and the excess cost of capital $\delta=0.1$. The rest of the parameters are as stated in footnote 6.

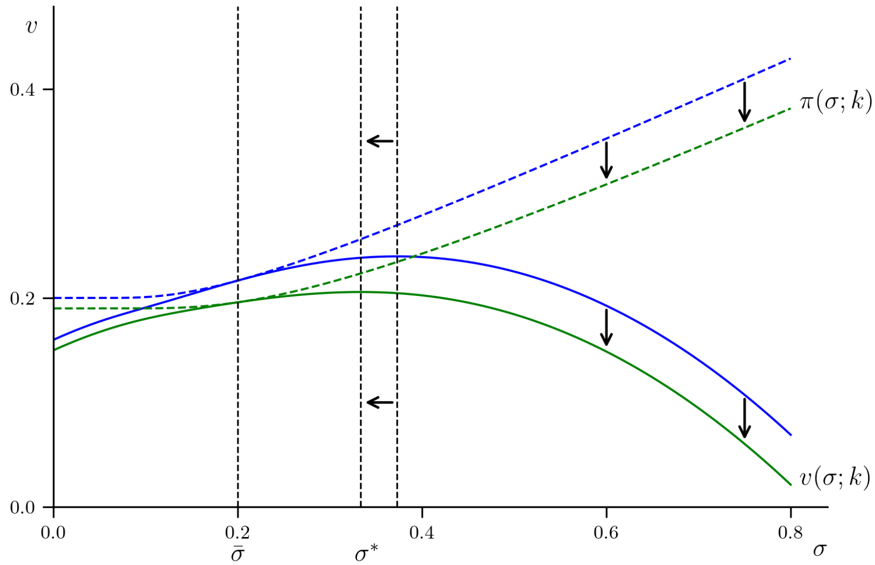

Figure 2. Risk-taking with a capital requirement

This figure plots the bank's expected payoff and objective function without (blue lines) and with (green lines) a capital requirement, showing its effect in reducing the bank's excess risk-taking.

The results obtained so far on the bank's incentives for excess risk-taking and the effect of capital requirements in ameliorating them are fairly standard, except for the specific model setup. The novel contribution of this paper is to show how this setup can be used to analyze the effect of bank supervision on risk-taking, which is done in the next section.

5 Bank supervision

I now introduce a bank supervisor that observes at $t = 1$ a *nonverifiable signal*

$$s = R + \varepsilon \tag{7}$$

of the final return R of the bank's investment, which is interpreted as the outcome of bank supervision. The noise term ε is assumed to be independent of the liquidation return L and the final return R, and has a distribution $N(0, \theta\sigma^2)$. The higher the value of θ the lower the quality (technically the precision) of the supervisory information. When $\theta = 0$ the supervisor observes at $t = 1$ the final return R that will obtain at $t = 2$, while when $\theta \to \infty$ the signal is completely uninformative.[9]

By assumptions (1) and (7), the joint probability distribution of the liquidation return L, the final return R, and the supervisory signal s is

$$\begin{bmatrix} L \\ R \\ s \end{bmatrix} \sim N\left(\overline{R}\begin{bmatrix} a \\ 1 \\ 1 \end{bmatrix}, \sigma^2\begin{bmatrix} b & c & c \\ c & 1 & 1 \\ c & 1 & 1+\theta \end{bmatrix}\right). \tag{8}$$

I assume that the supervisor closes the bank at $t = 1$ when, on the basis of the signal s, it assesses that the bank is *failing or likely to fail*, that is when the conditional expected final return $E(R \mid s)$ is smaller than the unit face value of the deposits.[10] By the properties of normal distributions (see (12) in Appendix A)

$$E(R \mid s) = \overline{R} + \frac{s - \overline{R}}{1+\theta} < 1$$

if and only if $s < \widehat{s}$, where

$$\widehat{s} = 1 - \theta(\overline{R} - 1) \tag{9}$$

is the liquidation threshold. Note that the higher the parameter θ that characterizes the variance of the noise in the supervisory information, the lower the liquidation threshold $\widehat{s}$ and hence the softer the supervisor.[11]

I assume that if the bank is declared failing or likely to fail, the supervisor uses the liquidation proceeds to cover deposit insurance payouts, so the bank gets a zero payoff upon

[9] It should be noted that the signal s is not about the choice of σ by the bank at $t = 0$, but about the consequences of this choice in terms of a particular value of the final return R at $t = 2$. Arguably, this provides a better approximation to the actual behavior of bank supervisors who typically do not act when high risk-taking leads to expected high return realizations.

[10] This assumption is grounded on the behavior of bank supervisors. For example, according to the Banking Supervision guidelines of the European Central Bank, "there are four reasons why a bank can be declared failing or likely to fail: (i) it no longer fulfils the requirements for authorization by the supervisor, (ii) it has more liabilities than assets, (iii) it is unable to pay its debts as they fall due, and (iv) it requires extraordinary financial public support. At the time of declaring a bank as failing or likely to fail, one of the above conditions must be met or be likely to be met." In our setup, the supervisor assesses that the bank has "more liabilities than assets" whenever $E(R \mid s) < 1$.

[11] An alternative rule would be to close the bank when $s < \widetilde{s}$, where $\widetilde{s}$ is defined by $\Pr(R < 1 \mid \widetilde{s}) = 1 - \omega$ and ω is a given confidence level (such as 95%). One can show that this rule simplifies to $s < \widetilde{s}$, where $\widetilde{s} = \widehat{s} + \sigma\sqrt{\theta(1+\theta}\Phi^{-1}(\omega)$. Note the unlike the failing or likely to fail threshold $\widehat{s}$, the threshold $\widetilde{s}$ depends on the (unobserved) risk σ chosen by the bank.

liquidation.[12] Under this assumption, the bank's objective function becomes

$$v(\sigma;\widehat{s}) = \pi(\sigma;\widehat{s}) - c(\sigma),$$

where

$$\pi(\sigma;\widehat{s}) = E[R-1 \mid R \geq 1 \text{ and } s \geq \widehat{s}]\,\Pr(R \geq 1 \text{ and } s \geq \widehat{s})$$

is the bank's expected payoff when the supervisor liquidates the bank upon observing a signal $s < \widehat{s}$.

Using the properties of truncated bivariate normal distributions (see (15) in Appendix A) we have

$$\begin{aligned}\pi(\sigma;\widehat{s}) \;&=\; (\overline{R}-1)\Phi\left(\frac{\overline{R}-1}{\sigma},\frac{\overline{R}-\widehat{s}}{\sigma\sqrt{1+\theta}};\frac{1}{\sqrt{1+\theta}}\right)+\sigma\phi\left(\frac{\overline{R}-1}{\sigma}\right)\Phi\left(\frac{1-\widehat{s}}{\sigma\sqrt{\theta}}\right)\\ &\quad+\frac{\sigma}{\sqrt{1+\theta}}\phi\left(\frac{\overline{R}-\widehat{s}}{\sigma\sqrt{1+\theta}}\right)\Phi\left(\frac{\widehat{s}-1+\theta(\overline{R}-1)}{\sigma\sqrt{\theta(1+\theta)}}\right),\end{aligned}$$

where $\Phi(\cdot,\cdot;\rho)$ is the cumulative distribution function of a standard bivariate normal distribution with correlation coefficient ρ.[13] Substituting $\widehat{s} = 1-\theta(\overline{R}-1)$ from (9) into this expression then gives

$$\begin{aligned}\pi(\sigma;\theta) \;&=\; (\overline{R}-1)\Phi\left(\frac{\overline{R}-1}{\sigma},\frac{\sqrt{1+\theta}(\overline{R}-1)}{\sigma};\frac{1}{\sqrt{1+\theta}}\right)+\sigma\phi\left(\frac{\overline{R}-1}{\sigma}\right)\Phi\left(\frac{\sqrt{\theta}(\overline{R}-1)}{\sigma}\right)\\ &\quad+\frac{\sigma}{2\sqrt{1+\theta}}\phi\left(\frac{\sqrt{1+\theta}(\overline{R}-1)}{\sigma}\right).\end{aligned}$$

Figure 3 shows the bank's expected payoff and objective function without (in blue) and with (in red) a supervisor that uses the failing or likely to fail threshold $\widehat{s}$.[14] The effect of bank supervision is similar to that of bank capital regulation analyzed in Section 4: both functions are shifted down, in this case because of the negative effect of the possible early

[12] This assumption is, again, grounded on the behavior of bank supervisors, who start their intervention by removing the ownership of the existing shareholders (and firing the management). Note that for signals s below the threshold $\widehat{s}$, assumption (2) implies that the expected liquidation proceeds satisfy $E(L \mid s) < E(L \mid \widehat{s}) = a\overline{R} - c(\overline{R}-1) < 1$.

[13] The correspondence between the variables in the expression in the Appendix and the variables in the model is as follows: $x_1 = R-1$, $x_2 = s$, $\mu_1 = \overline{R}-1, \mu_2 = \overline{R}$, $\sigma_1 = \sigma$, $\sigma_2 = \sigma\sqrt{1+\theta}$, $\rho = 1/\sqrt{1+\theta}$, $\widehat{x}_1 = 0$, and $\widehat{x}_2 = \widehat{s}$.

[14] In this figure I set $\theta = 1$. The rest of the parameters are as stated in footnote 6.

liquidation on the bank's expected payoff. Moreover, as will be explained below, the bank chooses a lower level of risk compared to the laissez-faire case.

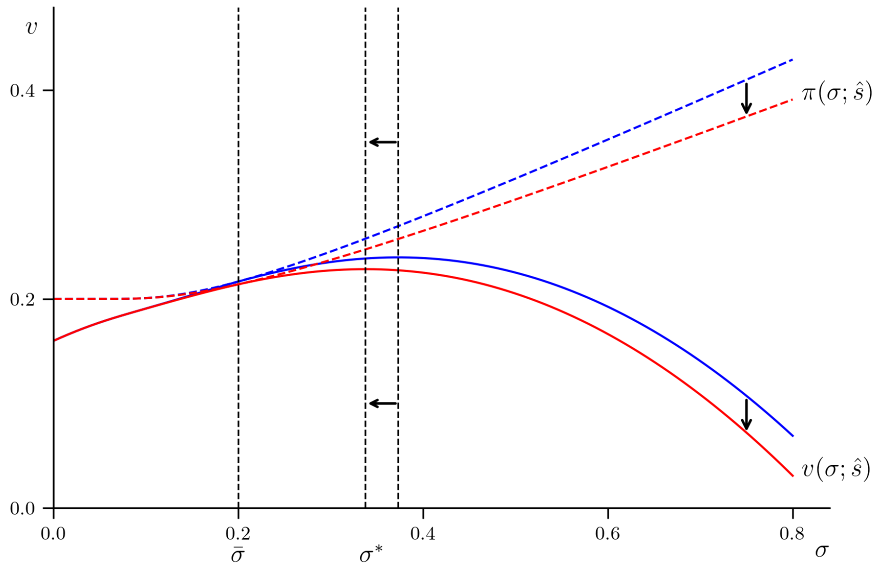

Figure 3. Risk-taking with bank supervision

This figure plots bank's expected payoff and objective function without (blue lines) and with (red lines) a supervisor that uses the failing or likely to fail threshold, showing its effect in reducing the bank's excess risk-taking.

Let us now define

$$\sigma^*(\theta) = \arg\max_{\sigma}[\pi(\sigma;\theta) - c(\sigma)]$$

as the bank's choice of risk when the parameter that characterizes the variance of the noise in the supervisory signal is θ. To analyze the effect of supervisory noise on risk-taking, it is useful to start with the limit cases $\theta = 0$ and $\theta \to \infty$.

When $\theta = 0$ the supervisor observes at $t = 1$ the value of the final return R that will obtain at $t = 2$. Moreover, the liquidation threshold defined in (9) becomes $\widehat{s} = 1$, which coincides with the value of the bank's liabilities. This means that the bank would be liquidated by the supervisor at $t = 1$ if and only if the bank would fail at $t = 2$. Supervision would not have any effect on the bank's choice of risk, which would be identical to the one in laissez-faire.

When $\theta \to \infty$ the liquidation threshold defined in (9) satisfies $\widehat{s} \to -\infty$, which means that the supervisor would never liquidate the bank at $t = 1$. Once again, supervision would not have any effect on the bank's choice of risk, which would be identical to the one in laissez-faire.

Thus, in the two limit cases of perfectly informative ($\theta = 0$) and perfectly uninformative ($\theta \to \infty$) supervisory information the bank's choice of risk is the same as in laissez-faire. At the same time, Figure 5 shows that for a positive value of θ the bank chooses a lower level of risk compared to the laissez-faire case, so $\sigma^*(\theta)$ cannot be monotonic.

Figure 4 plots the function $\sigma^*(\theta)$, showing that the effect of parameter θ (in the horizontal axis) on the bank's choice of risk σ^* (in the vertical axis) is U-shaped, first decreasing and then increasing, with $\sigma^*(0)$ and $\lim_{\theta\to\infty}\sigma^*(\theta)$ equal to the choice of risk in laissez-faire. Thus, when θ is small, higher noise in the supervisory information leads to lower lower risk-taking. Conversely, when θ is large, higher noise in the supervisory information leads to higher risk-taking.

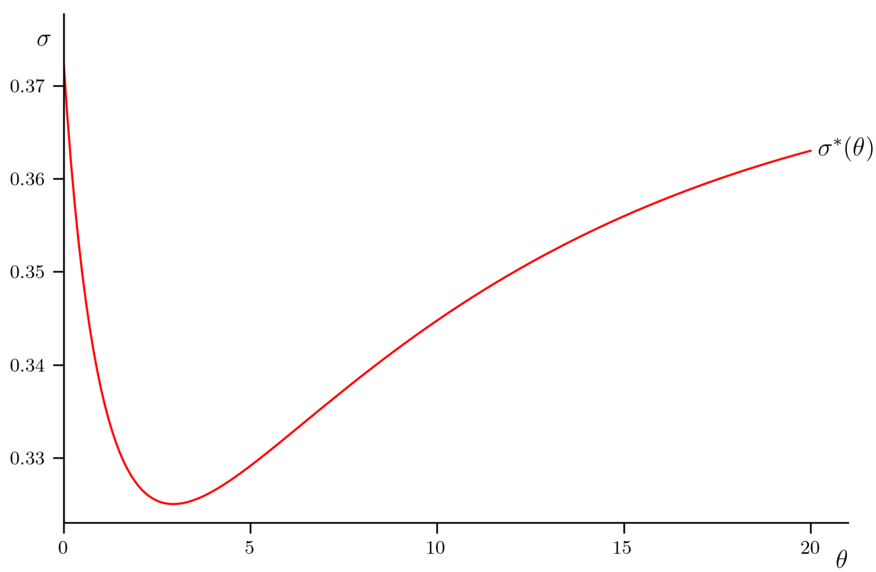

Figure 4. Effect of supervision on risk-taking

This figure shows the relationship between the variance of noise parameter in the supervisor's signal (in the horizontal axis) and the bank's choice of risk (in the vertical axis).

To understand why the disciplining effects of bank supervision come from the fact that supervisory information is noisy, consider Figure 5. The four panels plot regions with the final return R in the horizontal axis and the noise in the supervisory signal ε in the vertical axis. Panel A shows the region with $R < 1$ where the bank would fail at $t = 2$, and Panel B the region with $s = R + \varepsilon < \widehat{s}$ where the bank is liquidated by the supervisor at $t = 1$. The key intersection region in Panel C shows the combinations of R and ε for which the bank is liquidated at $t = 1$ but it would have been solvent at $t = 2$. In this region, the supervisor makes a type I (or false positive) error, liquidating the bank when it should have been allowed to continue. This error is due to a large negative realization of the noise term ε, which worsens the signal s observed by the supervisor. Finally, Panel D shows the effect of an increase in the variance of the noise parameter θ, which according to (9) reduces the liquidation threshold $\widehat{s}$.

The result in Figure 4 that $\sigma^*(\theta) < \sigma^*(0)$ for $\theta > 0$ can now be explained by the fact that the key intersection region in Panel C of Figure 5 disappears when $\theta = 0$, because the failure region coincides with the liquidation region. Since the bank's choice of risk for $\theta = 0$ is the same as in laissez-faire, for $\theta > 0$ the bank has an incentive to choose a lower value of σ to reduce the probability of falling into the intersection region, and being liquidated by the supervisor. Moreover, this argument implies that $\sigma^*(\theta)$ has to be decreasing for small values of θ.

With regard to the effects of changes in θ, note that an increase in θ has two effects. First, as shown in Panel D of Figure 5, it reduces the size of the key intersection region, where the bank is erroneously liquidated at $t = 1$. Second, it makes it more likely that large negative values of the noise ε in the supervisory signal s are realized. The first effect tends to increase the risk σ chosen by the bank, while the second effect goes in the opposite direction. Our results in Figure 4 show that the first (second) effect dominates when θ is large (small).

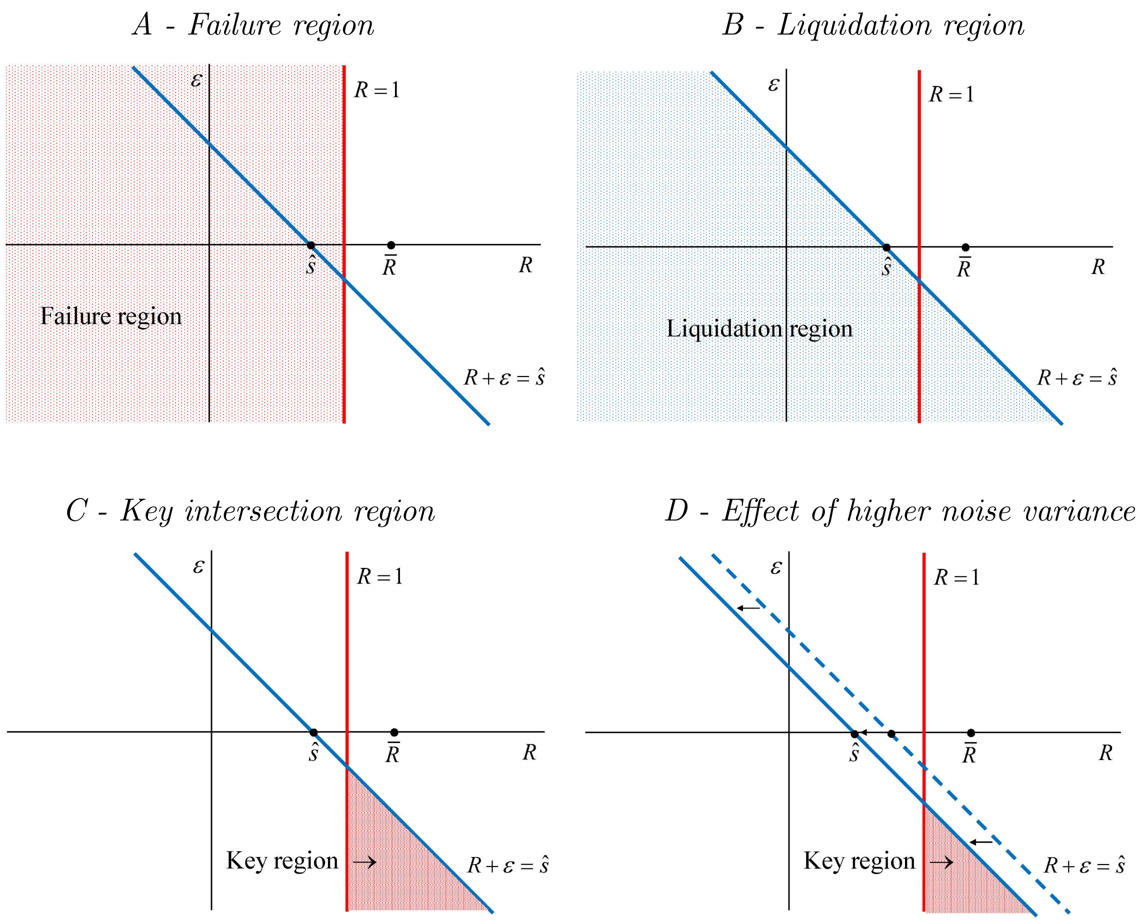

Figure 5. Failure and liquidation regions

This figure plots regions in a space with the final bank return in the horizontal axis and the noise in the supervisory signal in the vertical axis. Panel A shows the region where the bank would fail at $t = 2$. Panel B shows the region where the bank would be liquidated by the supervisor at $t = 1$. Panel C shows the key region where a solvent bank at $t = 2$ would be liquidated at $t = 1$. Panel D shows the effect on the region in Panel C of an increase in the variance of the noise parameter.

Summing up, this section has analyzed the bank's choice of risk when there is a supervisor that observes a signal s on the final return R of the bank's investment, liquidating the bank when the signal indicates that the bank is failing or likely to fail. The presence of a supervisor leads the bank to choose a lower level of risk, compared to laissez-faire. Thus, *supervision is effective in ameliorating the bank's risk-shifting incentives*. Moreover, for sufficiently high quality of the supervisory information, *higher noise in the supervisory* *information is conducive to lower risk-taking.*

6 Bank supervision with capital requirements

This section analyzes the effect of supervision when the bank is subject to a minimum capital requirement k. In this case the supervisor liquidates the bank at $t = 1$ when it observes a signal s such that the expected final return $E(R \mid s)$ is smaller than the face value of the deposits, which is now $1 - k$. Thus, we have

$$E(R \mid s) = \overline{R} + \frac{s - \overline{R}}{1+\theta} < 1 - k$$

if and only if $s < \widehat{s}(k)$, where

$$\widehat{s}(k) = 1 - k - \theta[\overline{R} - (1-k)] \tag{10}$$

is the new liquidation threshold. Since the threshold $\widehat{s}(k)$ is decreasing in k, the range of signals for which the bank is assessed to be failing or likely to fail is smaller. In other words, the supervisor becomes softer when the bank has a capital buffer. As before, the higher the parameter θ that characterizes the variance of the noise of the supervisory information, the lower the liquidation threshold $\widehat{s}(k)$ and hence the softer the supervisor.[15]

The bank's objective function now becomes

$$v(\sigma; k, \widehat{s}(k)) = \pi(\sigma; k, \widehat{s}(k)) - c(\sigma),$$

where

$$\pi(\sigma; k, \widehat{s}(k)) = E[R-(1-k) \mid R \geq 1-k \text{ and } s \geq \widehat{s}(k)] \Pr(R \geq 1-k \text{ and } s \geq \widehat{s}(k)) - (1+\delta)k.$$

is the bank's expected payoff when it is subject to a capital requirement k and the supervisor uses the failing or likely to fail threshold $\widehat{s}(k)$.

[15] This effect could in principle counter the comparative advantage of insured deposits, questioning the assumption that the capital requirement is binding, but the numerical results show that, for any quality of the supervisory information, the bank prefers to have as little capital as possible.

Using the properties of truncated bivariate normal distributions (see (15) in Appendix A) we have

$$\begin{aligned}\pi(\sigma;k,\widehat{s}(k)) &= [\overline{R}-(1-k)]\Phi\left(\frac{\overline{R}-(1-k)}{\sigma},\frac{\overline{R}-\widehat{s}(k)}{\sigma\sqrt{1+\theta}};\frac{1}{\sqrt{1+\theta}}\right)\\ &+\sigma\phi\left(\frac{\overline{R}-(1-k)}{\sigma}\right)\Phi\left(\frac{1-k-\widehat{s}(k)}{\sigma\sqrt{\theta}}\right)\\ &+\frac{\sigma}{\sqrt{1+\theta}}\phi\left(\frac{\overline{R}-\widehat{s}(k)}{\sigma\sqrt{1+\theta}}\right)\Phi\left(\frac{\widehat{s}(k)-(1-k)+\theta[\overline{R}-(1-k)]}{\sigma\sqrt{\theta(1+\theta)}}\right)-(1+\delta)k,\end{aligned}$$

where $\Phi(\cdot,\cdot;\rho)$ is the cumulative distribution function of a standard bivariate normal distribution with correlation coefficient ρ.[16] Substituting $\widehat{s}(k)=1-k-\theta[\overline{R}-(1-k)]$ from (10) into this expression then gives

$$\begin{aligned}\pi(\sigma;k,\theta) &= [\overline{R}-(1-k)]\Phi\left(\frac{\overline{R}-(1-k)}{\sigma},\frac{\sqrt{1+\theta}[\overline{R}-(1-k)]}{\sigma};\frac{1}{\sqrt{1+\theta}}\right)\\ &+\sigma\phi\left(\frac{\overline{R}-(1-k)}{\sigma}\right)\Phi\left(\frac{\sqrt{\theta}[\overline{R}-(1-k)]}{\sigma}\right)\\ &+\frac{\sigma}{2\sqrt{1+\theta}}\phi\left(\frac{\sqrt{1+\theta}[\overline{R}-(1-k)]}{\sigma}\right)-(1+\delta)k.\end{aligned}$$

Let us now define

$$\sigma^*(k,\theta)=\arg\max_{\sigma}[\pi(\sigma;k,\theta)-c(\sigma)]$$

as the bank's choice of risk when it is subject to a capital requirement k and the parameter that characterizes the variance of the noise in the supervisory signal is θ.

Figure 6 plots the bank's choice of risk $\sigma^*(k,\theta)$ as a function of the capital requirement k (in the horizontal axis) and the noise parameter θ (in the vertical axis). By our previous results, $\sigma^*(0,0)$ corresponds to the bank's choice of risk in laissez-faire. The results show that capital requirements and noisy supervision are effective in reducing the bank's risk-taking incentives, but that capital requirements are more powerful tools. They also show that, for any given quality of the supervisory information, one could target a desired level

[16] The correspondence between the variables in the expression in the Appendix and the variables in the model is as follows: $x_1=R-(1-k)$, $x_2=s$, $\mu_1=\overline{R}-(1-k)$, $\mu_2=\overline{R}$, $\sigma_1=\sigma$, $\sigma_2=\sigma\sqrt{1+\theta}$, $\rho=1/\sqrt{1+\theta}$, $\widehat{x}_1=0$, and $\widehat{x}_2=\widehat{s}(k)$.

of safety and soundness by adjusting the capital requirement, with lower requirements for intermediate values of the noise parameter θ. This is consistent with our previous results on the U-shaped relationship between the quality of the supervisory information and the bank's choice of risk.

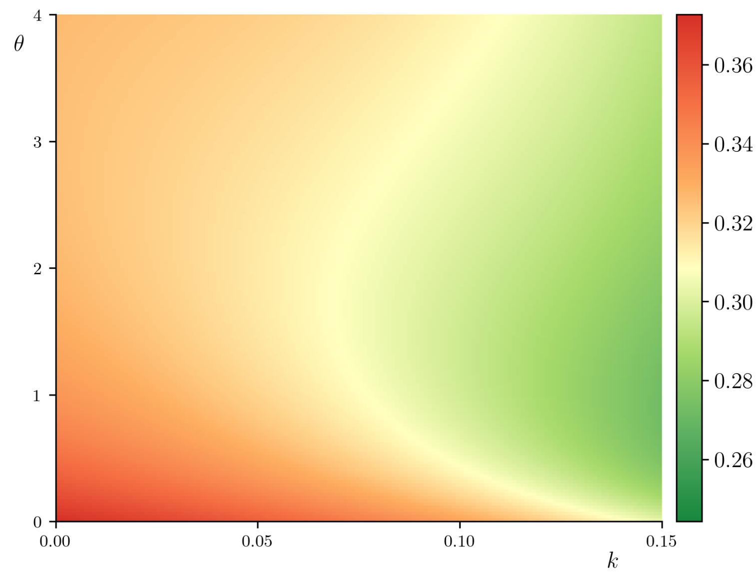

Figure 6. Effect of regulation and supervision on risk-taking

This figure shows the bank's choice of risk as a function of the capital requirement (in the horizontal axis) and the parameter that characterizes the variance of the noise in the supervisory signal (in the vertical axis).

7 Welfare analysis

I next turn to the welfare analysis of bank regulation and supervision. The social planner chooses the capital requirement k and the parameter θ that characterizes the quality of the supervisory information, instructing the supervisor to use the failing or likely to fail rule as stated in Section 6. I assume that bank supervision is costless and that deposit insurance is funded with lump sum taxation. I also assume that bank's (orderly) liquidation by the supervisor at $t = 1$ does not entail any externalities, but the bank's failure at $t = 2$ entails a social cost $\kappa > 0$.

By the properties of truncated bivariate normal distributions (see (14) in Appendix A), the probability that this cost will be incurred is given by

$$p^* = p(k,\theta) = \Pr[R < 1-k \text{ and } s \geq \widehat{s}] = \Phi\left(\frac{1-k-\overline{R}}{\sigma^*}, \frac{\overline{R}-\widehat{s}}{\sigma^*\sqrt{1+\theta}}; -\frac{1}{\sqrt{1+\theta}}\right),$$

where $\widehat{s} = \widehat{s}(k)$ is the liquidation threshold given by (10) and $\sigma^* = \sigma^*(k,\theta)$ is the bank's choice of risk.

Under these assumptions, social welfare $w(k,\theta)$ is simply the expected payoff of the investment, taking into account the liquidation threshold of the supervisor, minus the sum of bank's cost of risk-taking $c(\sigma^*)$, the excess cost of bank capital δk, the expected cost of bank failure $p^*\kappa$, and the unit cost of the investment, that is

$$w(k,\theta) = E(R \mid s \geq \widehat{s}) \Pr(s \geq \widehat{s}) + E(L \mid s < \widehat{s}) \Pr(s < \widehat{s}) - c(\sigma^*) - \delta k - p^*\kappa - 1. \quad (11)$$

By the properties of truncated bivariate normal distributions (see (16) and (17) in Appendix A) we have

$$E(R \mid s \geq \widehat{s}) \Pr(s \geq \widehat{s}) = \overline{R}\Phi\left(\frac{\overline{R}-\widehat{s}}{\sigma^*\sqrt{1+\theta}}\right) + \frac{\sigma^*}{\sqrt{1+\theta}}\phi\left(\frac{\overline{R}-\widehat{s}}{\sigma^*\sqrt{1+\theta}}\right),$$

and

$$E(L \mid s < \widehat{s}) \Pr(s < \widehat{s}) = a\overline{R}\Phi\left(\frac{\widehat{s}-\overline{R}}{\sigma^*\sqrt{1+\theta}}\right) - \frac{\sigma^* c}{\sqrt{1+\theta}}\phi\left(\frac{\widehat{s}-\overline{R}}{\sigma^*\sqrt{1+\theta}}\right).$$

Substituting the threshold $\widehat{s}(k)$ from (10) into these expressions, plugging them into (11) and rearranging gives

$$\begin{aligned} w(k,\theta) &= a\overline{R} + (1-a)\overline{R}\Phi\left(\frac{\sqrt{1+\theta}[\overline{R}-(1-k)]}{\sigma^*}\right) \\ &\quad + \frac{\sigma(1-c)}{\sqrt{1+\theta}}\phi\left(\frac{\sqrt{1+\theta}[\overline{R}-(1-k)]}{\sigma^*}\right) - c(\sigma^*) - \delta k - p^*\kappa - 1. \end{aligned}$$

Figure 7 plots social welfare $w(k,\theta)$ as a function of the capital requirement k (in the horizontal axis) and the parameter θ that characterizes the variance of the noise in the supervisory signal (in the vertical axis). It should be noted that $w(0,0)$ does not correspond to social welfare in laissez-faire, even though $\sigma^*(0,0)$ is the bank's choice of risk in laissez-faire. This is because under perfect information ($\theta = 0$) and zero capital requirements

($k = 0$), the supervisor liquidates the bank whenever $s = R < 1 = \widehat{s}(0)$, saving the cost of bank failure $p^* \kappa = \Pr(R < 1)\kappa$, but incurring a potential liquidation cost given by the expected value of the difference between the final return and the conditional liquidation return, that is

$$E[R - E(L \mid R) \mid R < 1] \Pr(R < 1).$$

By the properties of normal distributions (see (12) in Appendix A), this expression simplifies to

$$(1 - c)E[R - R^* \mid R < 1] \Pr(R < 1),$$

where R^* is given by (2). Since I have assumed that $R^* < 1$, there is a range $(R^*, 1)$ of values of the final return R for which the difference $R - R^*$ is positive, and a range $(-\infty, R^*)$ for which it is negative, so the sign of this term is ambiguous. To make the results easier to interpret, in Figure 7 I have set the social cost of bank failure κ such that $w(0, 0)$ equals the social welfare in laissez-faire.[17]

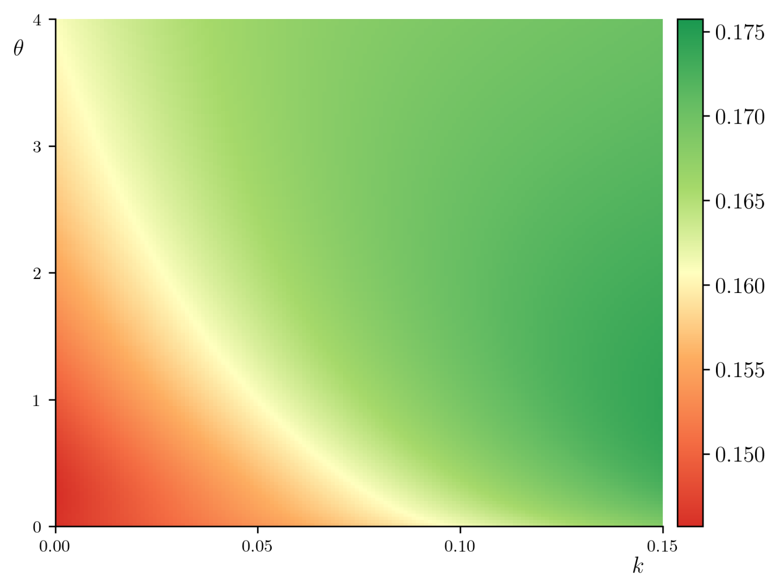

Figure 7. Welfare effects of regulation and supervision

This figure shows social welfare as a function of the capital requirement (in the horizontal axis) and the parameter that characterizes the variance of the noise in the supervisory signal (in the vertical axis).

[17]This requires to set $\kappa = 0.08339$.

The results show that, for any given quality of the supervisory information, bank capital regulation improves welfare, although beyond certain level the effect is almost nil. The results also show that, for any level of the capital requirement, higher noise in the supervisory information (within a reasonable range) improves welfare, although for high capital requirements the effect of supervision is essentially zero.

The conclusion that follows from these results is that, from a social welfare perspective, bank capital regulation is a better tool for addressing the bank's risk taking incentives. Noisy bank supervision can help but only when capital requirements are low.

8 Concluding remarks

This paper presents a model of the interaction between a bank that chooses the risk of its investment portfolio and a supervisor that collects nonverifiable information on the solvency of the bank and, based on of this information, may decide on its early liquidation. The paper characterizes the liquidation decision of the supervisor and the risk-taking decision of the bank. In line with recent empirical literature, the paper shows that supervision is effective in ameliorating the bank's risk-taking incentives, and that higher noise in the supervisory information may be conducive to lower risk-taking. The paper also analyzes the interaction between regulation and supervision, showing that the capital requirements and noisy supervision are effective in reducing the bank's risk-taking incentives, but that capital requirements are more powerful tools. Finally, the paper provides a welfare analysis of regulation and supervision, showing that noisy supervision improves improve welfare, especially when capital requirements are low, but that bank capital regulation is a better tool for addressing the bank's risk-taking incentives.

I would like to conclude with a few remarks. First, the model assumes that the supervisor follows a failing or likely to fail rule to decide on liquidation. But it could be used to analyze softer rules. In particular, for the model without capital requirements (and similarly for the model with them), one could assume that the supervisor closes the bank at $t = 1$ when

$$E(R \mid s) = \overline{R} + \frac{s - \overline{R}}{1 + \theta} < 1 - \nu,$$

where ν is a parameter that captures a bias against liquidation. This implies a lower threshold for liquidation given by

$$\widehat{s}(\nu) = 1 - \nu - \theta(\overline{R} - 1 + \nu).$$

Figure 8 shows that, for any quality of the supervisory information, a softer supervisor that uses the threshold $\widehat{s}(\nu)$ leads to higher risk-taking. Thus, the model provides a rationale for some of the empirical results presented in Section 1.[18]

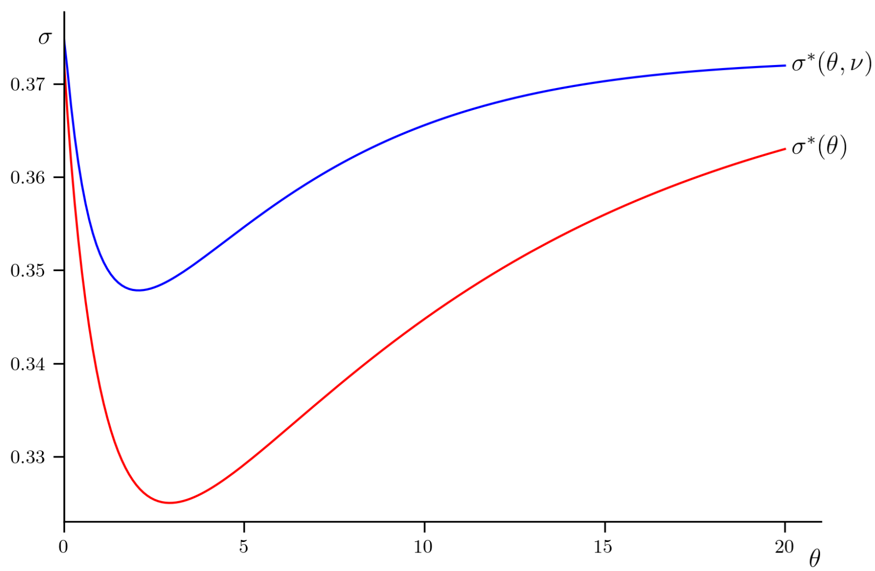

Figure 8. Effect of a softer supervisor

This figure shows the relationship between the variance of noise parameter in the supervisor's signal (in the horizontal axis) and the bank's choice of risk (in the vertical axis) for a tougher (red line) and a softer (blue line) supervisor.

Second, the closure of a bank by a supervisor using the failing or likely to fail need not imply liquidation, since it could lead to the transfer of the bank to another authority (such as the Federal Deposit Insurance Corporation in the US or the Single Resolution Board in

[18] In this figure I take $\nu = 0.05$. The rest of the parameters are as stated in footnote 6.

the European Union) that would decide between resolution and liquidation. In this case the bank would only be liquidated when it is efficient to do so, that is when the conditional expected liquidation return is greater than the conditional expected final return. Either way, the bank shareholders would be wiped out (and the management fired), which is key for controlling risk-taking incentives.

Third, there is an additional role of supervision that it is not taken into account in these results, namely ensuring compliance with regulation. Our focus is on the assessment of solvency by gathering confidential supervisory information and the use of this information to take corrective actions (in our case liquidation). But the very significant effects of regulation on risk-taking incentives require that it be properly enforced, which is a key role of bank supervision.

Finally, the model is static, but one could easily construct a dynamic version with endogenous charter values. As it is well known in the banking literature (see, for example, Repullo, 2004), charter values (the discounted value of future rents) provide incentives for prudent bank behavior, so one would expect that the effect of both regulation and supervision be more muted.

Appendix

A Some useful properties of normal distributions

This Appendix summarizes four properties of the expectation of normal random variables used in the paper.[19] Consider a pair (x_1, x_2) of random variables with

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \right),$$

where $-1 < \rho < 1$.

The first property is the expectation of x_1 conditional on x_2, which is

$$E(x_1 \mid x_2) = \mu_1 + \frac{\rho\sigma_1}{\sigma_2}(x_2 - \mu_2). \tag{12}$$

The second property is the expectation of x_1 conditional on $x_1 \geq \widehat{x}_1$, which is

$$E(x_1 \mid x_1 \geq \widehat{x}_1) = \mu_1 + \sigma_1 \frac{\phi\left(\frac{\mu_1 - \widehat{x}_1}{\sigma_1}\right)}{\Phi\left(\frac{\mu_1 - \widehat{x}_1}{\sigma_1}\right)}.$$

Since

$$\Pr(x_1 \geq \widehat{x}_1) = \Phi\left(\frac{\mu_1 - \widehat{x}_1}{\sigma_1}\right),$$

this implies

$$E(x_1 \mid x_1 \geq \widehat{x}_1) \Pr(x_1 \geq \widehat{x}_1) = \mu_1 \Phi\left(\frac{\mu_1 - \widehat{x}_1}{\sigma_1}\right) + \sigma_1 \phi\left(\frac{\mu_1 - \widehat{x}_1}{\sigma_1}\right). \tag{13}$$

The third property is the expectation of x_1 conditional on $x_1 \geq \widehat{x}_1$ and $x_2 \geq \widehat{x}_2$, which is

$$\begin{aligned} E(x_1 \mid x_1 \geq \widehat{x}_1 \text{ and } x_2 \geq \widehat{x}_2) \quad = \quad & \mu_1 + \frac{\sigma_1}{F(\widehat{x}_1, \widehat{x}_2)} \phi\left(\frac{\widehat{x}_1 - \mu_1}{\sigma_1}\right) \Phi\left(\frac{\rho\frac{\widehat{x}_1 - \mu_1}{\sigma_1} - \frac{\widehat{x}_2 - \mu_2}{\sigma_2}}{\sqrt{1-\rho^2}}\right) \\ & + \frac{\rho\sigma_1}{F(\widehat{x}_1, \widehat{x}_2)} \phi\left(\frac{\widehat{x}_2 - \mu_2}{\sigma_2}\right) \Phi\left(\frac{\rho\frac{\widehat{x}_2 - \mu_2}{\sigma_2} - \frac{\widehat{x}_1 - \mu_1}{\sigma_1}}{\sqrt{1-\rho^2}}\right) \end{aligned}$$

where

$$F(\widehat{x}_1, \widehat{x}_2) = \Pr(x_1 \geq \widehat{x}_1 \text{ and } x_2 \geq \widehat{x}_2) = \Phi\left(\frac{\mu_1 - \widehat{x}_1}{\sigma_1}, \frac{\mu_2 - \widehat{x}_2}{\sigma_2}; \rho\right) \tag{14}$$

[19] See the Appendix in Maddala (1983). He cites the work of Rosenbaum (1961) for the moments of the truncated bivariate normal distribution.

and $\Phi(\cdot,\cdot;\rho)$ is the cumulative distribution function of a standard bivariate normal distribution with correlation coefficient ρ. This implies

$$\begin{aligned}
&E(x_1 \mid x_1 \geq \widehat{x}_1 \text{ and } x_2 \geq \widehat{x}_2)\Pr(x_1 \geq \widehat{x}_1 \text{ and } x_2 \geq \widehat{x}_2) \\
&\quad = \mu_1 \Phi\left(\frac{\mu_1 - \widehat{x}_1}{\sigma_1}, \frac{\mu_2 - \widehat{x}_2}{\sigma_2}; \rho\right) + \sigma_1 \phi\left(\frac{\widehat{x}_1 - \mu_1}{\sigma_1}\right) \Phi\left(\frac{\rho \frac{\widehat{x}_1 - \mu_1}{\sigma_1} - \frac{\widehat{x}_2 - \mu_2}{\sigma_2}}{\sqrt{1-\rho^2}}\right) \\
&\quad + \rho\sigma_1 \phi\left(\frac{\widehat{x}_2 - \mu_2}{\sigma_2}\right) \Phi\left(\frac{\rho \frac{\widehat{x}_2 - \mu_2}{\sigma_2} - \frac{\widehat{x}_1 - \mu_1}{\sigma_1}}{\sqrt{1-\rho^2}}\right) \qquad (15)
\end{aligned}$$

The fourth property specializes the previous result for the case where $\widehat{x}_1 = -\infty$, which gives

$$E(x_1 \mid x_2 \geq \widehat{x}_2)\Pr(x_2 \geq \widehat{x}_2) = \mu_1 \Phi\left(\frac{\mu_2 - \widehat{x}_2}{\sigma_2}\right) + \rho\sigma_1 \phi\left(\frac{\widehat{x}_2 - \mu_2}{\sigma_2}\right). \qquad (16)$$

From here it follows that

$$\begin{aligned}
E(x_1 \mid x_2 < \widehat{x}_2)\Pr(x_2 < \widehat{x}_2) &= E(x_1 \mid -x_2 > -\widehat{x}_2)\Pr(-x_2 > -\widehat{x}_2) \\
&= \mu_1 \Phi\left(\frac{\widehat{x}_2 - \mu_2}{\sigma_2}\right) - \rho\sigma_1 \phi\left(\frac{\mu_2 - \widehat{x}_2}{\sigma_2}\right). \qquad (17)
\end{aligned}$$

B Proofs

Proof of Proposition 1 To prove that there is a solution to the bank's choice of risk note that

$$\Phi\left(\frac{\overline{R}-1}{\sigma}\right) < 1 \text{ and } \phi\left(\frac{\overline{R}-1}{\sigma}\right) < \phi(0)$$

imply

$$v(\sigma) < \overline{R} - 1 + \sigma\phi(0) - \frac{\gamma}{2}(\sigma - \overline{\sigma})^2,$$

so we conclude

$$\lim_{\sigma\to\infty} v(\sigma) \leq \lim_{\sigma\to\infty} \left[\overline{R} - 1 + \sigma\phi(0) - \frac{\gamma}{2}(\sigma - \overline{\sigma})^2\right] = -\infty.$$

Moreover, since

$$v'(0) = \pi'(0) + \gamma\overline{\sigma} = \gamma\overline{\sigma} > 0,$$

it follows that there is a solution σ^* characterized by the first-order condition (4). □

Proof of Proposition 2 The proof is identical to that of Proposition 1. □

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